Prepared for
George C. Marshall Space Flight Center
Huntsville, Alabama

MULTIFREQUENCY ANTENNA BREAKDOWN

BY

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CONTRACT NAS8 - 5412
JUNE 5, 1963 - JUNE 4, 1964

BER

INTERIM REPORT JUNE, 1964

University of Alabama
Bureau of Engineering Research
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ACKNOWLEDGEMENT

This report has been prepared for the Instrumentation Branch, Astrionics Division, George C. Marshall Space Flight Center, Huntsville, Alabama, under Contract NAS8-5412.

The author wishes to acknowledge the valuable assistance of Professor O. P. McDuff, formerly Project Co-Director and now on leave of absence, in the performance of the work reported herein. The administrative assistance of Professor L. A. Woodman, Director, Bureau of Engineering Research, is also gratefully acknowledged. The author also wishes to express his appreciation to Mr. J. W. Harper of the Astrionics Division, Marshall Space Flight Center, for many helpful technical discussions.

ABSTRACT

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The results of a theoretical and experimental investigation of high-altitude antenna voltage breakdown during the radiation of multiple signals from one antenna system are presented.

The experimental and theoretical work of other investigators is reported and applied to the problem of multifrequency breakdown.

Maximum and minimum breakdown power limits are developed theoretically.

Measurements of two- and three-frequency breakdown power are discussed.

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I. INTRODUCTION

The work to be discussed in this report consisted of a theoretical and experimental investigation of low-pressure antenna voltage breakdown during radiation of two or more signals of comparable amplitudes and frequency separation small compared to the individual frequencies.

The literature was searched and analyzed carefully in an effort to apply the work of other investigators to this problem. Considerable information was found concerning CW breakdown, and some published work dealt with pulse breakdown. Little was found, however, of direct applicability to multifrequency breakdown.

Next, work was done to develop limits on multifrequency breakdown and to develop equations for predicting breakdown. Both maximum and minimum power limits have been developed and are presented in this report. An equation for predicting breakdown is also presented. Unfortunately, the data necessary for proper use of this equation are not available in the published literature.

The third phase of the work discussed here was an experimental program to measure breakdown powers in a typical antenna radiating multiple signals. These measurements are presented in terms of breakdown power of a single frequency signal radiated from the same antenna.

Little was discovered in the course of these investigations to improve the multifrequency breakdown characteristics of antennas other than the already known techniques for improving the CW breakdown performance. However, values of multifrequency breakdown power for two, three, and, by extrapolation, four signals are presented in this report, and it is hoped that they will be useful to the antenna designer.

II. VOLTAGE BREAKDOWN PHENOMENA

Two excellent general references concerned with breakdown in non-radiating structures are by Brown[1] and Gould and Roberts[2]. It has been shown that primary ionization caused by electron motion is the principal source of electron production at high frequencies. The free electrons caused by external radiation, etc. gain energy from the radio-frequency field by having their ordered oscillatory motion changed to random motion on collision. This continues until an electron has gained sufficient energy to have an ionizing collision with a neutral particle. A gas discharge occurs when the gain in electron density by this ionization of the gas becomes equal to the loss of electrons by diffusion, recombination (of electrons with positive ions), and attachment (of electrons to neutral particles)[3].

When electron density increases to a sufficiently high value in the vicinity of an antenna transmission of a signal is substantially blocked. The density above which there is practically no transmission is the "plasma resonant density," n_p, which is equal to $10^{13}/\lambda^2$ electrons/cm^3 where λ is the wavelength in cm. At a concentration of $n_p/2$ signal absorption and reflection are negligible; when concentration is $2n_p$, no signal is passed, therefore, n_p may be considered a sharp upper limit for transmission.[4]

Electron Power Gain in an RF Field[1]

At the minimum fields experimentally determined for breakdown the energy imparted to an electron which does not collide with other particles is about 10^{-3} electron volts. This is insufficient to ionize, and it therefore follows that an electron gains energy until it has enough to ionize a neutral.

Assume the electric field and the electron velocity to vary exponentially.

$$E = E_o \varepsilon^{j\omega t} \tag{1}$$

$$v = V_o \varepsilon^{j\omega t} \tag{2}$$

where E_o is rms value.

The electron motion is given by

$$m \frac{dv}{dt} + (m\nu_c)v = -Ee \tag{3}$$

where ν_c is the collision frequency, m the electron mass and e the charge. With the substitution of eqs. (1) and (2) into (3) we obtain a phasor velocity

$$V_o = \frac{-eE_o}{j\omega m + m\nu_c} \quad . \tag{4}$$

The phasor current density is the product of this velocity, the electron charge, and the number of electrons per unit volume.

$$J = -neV_o = \frac{ne^2E_o}{j\omega m + m\nu_c}$$

$$= \frac{ne^2E_o}{m\omega} \; \frac{\frac{\nu_c}{\omega} - j}{\left(\frac{\nu_c}{\omega}\right)^2 + 1} \tag{5}$$

Power per unit volume transferred to the electrons is the product of the in-phase components of field strength and current density.

$$P = \frac{ne^2E_o^2}{m\omega} \; \frac{\nu_c/\omega}{\left(\frac{\nu_c}{\omega}\right)^2 + 1} = \frac{ne^2E_o^2}{m\nu_c} \; \frac{\nu_c^2}{\nu_c^2 + \omega^2} \tag{6}$$

An effective field intensity may be defined as

$$E_e^2 = E_o^2 \; \frac{\nu_c^2}{\nu_c^2 + \omega^2} \tag{7}$$

Then

$$P = \frac{ne^2 E_e^2}{m\nu_c} \tag{8}$$

Equation (8) gives the rate of increase of energy of the electrons in the field. This energy increases until utilized in exciting or ionizing collisions.

Margenau[5] points out that the term $m\nu_c$ of eq. (3) is often treated as an empirical parameter.

Breakdown Equation

The continuity equation for electrons is [6]

$$\frac{\partial n}{\partial t} = (\nu_i - \nu_a) n + S + \nabla^2 (nD) \tag{9}$$

where n is the electron density (electrons/cm^3)

ν_i is the ionization rate (ionizations/sec/electron)

ν_a is the attachment rate (attachments/sec/electron)

S is the ionization produced by an external source (electrons/cm^3 sec)

D is the diffusion coefficient for electrons (cm^2/sec)

Equation (9) neglects recombination as an electron-loss mechanism because of its small effect.[2] The last term in eq. (9) may be replaced by a term which describes diffusion in terms of an effective "diffusion length", L[7], so that eq. (9) becomes

$$\frac{\partial n}{\partial t} = (\nu_i - \nu_a) n + S - nD/L^2 \tag{10}$$

For parallel plate diffusion length L is equal to plate separation divided by π. In general, L is determined by the geometry of the structure (whether it is radiating or not is of no consequence).

In Eq. (10) electron loss by diffusion, represented by the last term, can be important for certain geometries. This is particularly true for CW breakdown. However, in pulsed breakdown the pulse may be on for such a short time that the diffusion losses are small. Then the field required for breakdown tends to be independent of the geometry of the breakdown structure.[6]

CW Breakdown

We neglect the term S in Eq. (10), ionization from an external source, as small in the vicinity of CW breakdown. Then Eq. (10) becomes for steady-state conditions[4]

$$\nu_i - \nu_a = D/L^2 \tag{11}$$

MacDonald[4], using measured data of other investigators, treats this CW breakdown very carefully. His results are presented here in Fig. 1 which shows the breakdown fields as a function of altitude for frequencies of 100 mc, and 3, 10, 20, and 35 Gc. Structural geometry is considered by setting $L = \lambda/2$ for the dashed curves (which might correspond to parallel plates separated by 4.7 cm at 10 Gc) and by making L infinitely large for the solid curves (approximated by a monopole antenna breaking down at the tip with no nearby structure to capture electrons).

Figure 1 shows very clearly the considerable advantage to be gained by using higher frequencies for transmission.

Gould and Roberts[2] also treat cω breakdown extensively, both theoretically and with measured data. Their work gives breakdown fields over a wide range of diffusion lengths. Scharfman and Morita[6] have made measurements on antennas as contrasted to the measurements of others dealing in the main with non-radiating structures. These authors consider monopole and aperture antenna breakdown, both CW and pulsed. Herlin and Brown[8,9] are a source of very complete data on CW breakdown. MacDonald[4] uses their data extensively.

Scharfman and Morita[6] also discuss transmission in the presence of an ionized medium, a plasma, associated with antenna reentry into the earths' atmosphere. In a short section they conclude that a positive space charge is developed around the antenna, retarding the high-velocity electrons and decreasing the electron diffusion rate. Thus the power-handling capability of the antenna is lowered by the ionized medium. In measurements, Scharfman and Morita simulated a reentry plasma (in electron density but not in electron temperature) by a dc glow discharge over the aperture of a slot antenna radiating at 373 mc. Without the plasma the antenna required 36 watts for breakdown. With a plasma of appropriate density (plasma frequency = $8900\sqrt{n}$ = RF frequency) the power for breakdown was reduced to 7 watts.

Pulse Breakdown

When a pulse is applied to an antenna in a low-pressure region the general form Eq. (10) must be used. Integrating Eq. (10) gives

$$\ln (n/n_o) = \int_o^t (\nu_i - \nu_a - D/L^2)\, dt \qquad (12)$$

where the effect of the external radiation, S, is to set the initial concentration, n_o, of electrons at zero time.

MacDonald[4] discusses pulse breakdown with the assumption that diffusion is unimportant. Then Eq. (12) becomes

$$\ln (n/n_o) = \int_o^t (\nu_i - \nu_a)\, dt = \int_o^t \nu\, dt \qquad (13)$$

where $\nu = \nu_i - \nu_a$. MacDonald's data are taken from the thorough CW measurements of Herlin and Brown.[8,9]

In Eq. (13), $\nu = \nu_i - \nu_a$ is a function only of the applied electric field. For a rectangular pulse ν is constant from time zero to time $t = \tau$, when the pulse is discontinued. Then Eq. (13) becomes

$$n = n_o \varepsilon^{\nu t} \tag{14}$$

If the concentration reaches the plasma resonant density, n_p, at the end of the pulse, practically the entire pulse will be transmitted. Thus

$$n_p = n_o \varepsilon^{\nu \tau} \tag{15}$$

allows the pulse to be transmitted. Eq. (15) may be solved for ν and from this the electric field may be found. A higher electric field would then not allow transmission of the complete pulse.

MacDonald uses a value of $n_o = 10^3$ electrons/cm^3 as typical, but points out that because of the exponential variation with time in Eq. (15) the maximum electric fields are not significantly affected if n_o reaches 2×10^5, a value which is possible in some regions of the ionosphere above 300,000 ft. Gould and Roberts[2] concur with this observation.

MacDonald presents his results in the form of curves giving the peak fields for which 90 percent of a rectangular pulse is transmitted. This is done for several pulse lengths and frequencies. Figure 2 was taken from his work and compares pulse breakdown fields with CW breakdown fields at 100 mc. Again it is noted that the pulse breakdown curve and the solid CW breakdown curve are based on the assumption that diffusion is unimportant. As expected, greater peak pulse power can be radiated before breakdown than is allowed for CW.

Gould and Roberts give additional single pulse breakdown data without making the assumption that diffusion is negligible. It is clear from their data that diffusion is important in parallel-plate breakdown only for the product of pressure (mmHg) and pulse length greater than 10^{-5}. At an altitude of 200,000 ft. this would give a pulse length greater than 50 microseconds. It is then apparent that MacDonald is justified in neglecting diffusion.

In the work on single pulse breakdown the assumption is made that the electrons created by one pulse are totally dissipated (by vehicle movement or the passage of time) before the next pulse occurs. If this is not true the electrons left over from one pulse will serve as the initial concentration for the next pulse and will lower the breakdown peak field from that of the single pulse.

For pulses which depart notably from the rectangular shape (Gould and Roberts discuss the quasi-rectangular case) Eq. (14) does not hold, and Eq. (13) may be integrated to give

$$n = n_o \varepsilon^{\int_o^t \nu dt} \tag{16}$$

If the time is sufficiently long the more general equation, derived from Eq. (12), must be used,

$$n = n_o \varepsilon^{\int_o^t (\nu_i - \nu_a - D/L^2)\, dt} \tag{17}$$

In the integration of Eq. (16), the pulse shape will generally be known and n_o assumed to sufficient accuracy. Then ν may be determined from measured data as a function of field strength and thus of time. The integration in Eq. (16) may be performed numerically. Eq. (16) then will yield the electron concentration as a function of time. If this reaches the plasma resonant density the remaining portion of the pulse will not be transmitted.

It should be noted that the problem of determining the amplitude of an odd-shaped pulse which may be transmitted without significant attenuation is quite difficult. It would conceivably be done best by assuming a peak amplitude and integrating in Eq. (16) to determine if the plasma resonant density of electrons were reached. Then the work would be repeated with a lower or higher value of pulse amplitude, with the process continued until the correct amplitude were obtained.

Figure 3 shows values of $\nu\lambda$ as functions of $E\lambda$ for various $p\lambda$ as a parameter. Wavelength is λ ; p is the pressure. This figure was adapted from MacDonald[4] who in turn developed it from the data of Herlin and Brown[8,9]. Figure 3 unfortunately does not extend to sufficiently low field strengths to be useful for anything except relatively short, high-amplitude pulses. In addition, diffusion is not taken into account in the data, and pulses of sufficient length, or repetitive pulses, cannot be handled in this manner.

The data necessary for determining breakdown fields for many non-rectangular pulses are lacking. MacDonald's data do not extend to low values of the field. Gould and Roberts discuss breakdown by non-rectangular pulses and use data other than that used by MacDonald. They obtain the ionization and attachment coefficients from Harrison and Geballe[10], and electron drift velocity data from Nielsen[11], Riemann[12], and Townsend[13]. Harrison and Geballe give their data in the form of

$$\frac{\alpha}{p} - \frac{\eta}{p}$$

where α is the first Townsend coefficient representing the number of ionizations made by an electron as it moves through one centimeter, η is a recombination coefficient representing the number of attachments per centimeter, and p is the pressure. They are related to the ionization and attachment coefficients previously discussed in this paper by the electron drift velocity.

$$\nu_i - \nu_a = \left(\frac{\alpha}{p} - \frac{\eta}{p} \right) p v_d \tag{18}$$

Unfortunately, the data of Harrison and Geballe do not extend to sufficiently low fields to allow consideration of long pulses of low amplitude. Their data extend in values of the ratio $E/_p$ from 25.0 to 60.0. By way of contrast, the value of $E/_p$ for CW breakdown at 100 mc and 200,000 ft. is 37.5 (MacDonald), while for breakdown for 1.0 microsecond pulses at the same frequency and altitude, $E/_p = 150$. It is apparent, then, that the studies of Gould and Roberts cannot be adapted to many problems of interest.

Gould and Roberts recognize this difficulty in their discussion of multi-pulse breakdown. They state that too little information is available on electron decay in the interval between pulses to allow satisfactory calculations.

For the multi-frequency breakdown phenomena considered in this report it will be shown later that the envelope of the electric field applied to the antenna can be considered as a repetitive pulse with a shape which departs greatly from the rectangular. It's amplitude varies from a maximum which is the sum of the individual fields applied to the antenna and changes through several smaller maxima (and minima) to a value which may be zero in some cases. In theory the solution of Eqs. (16) or (17) to obtain breakdown fields seems attractive (although long), but in practice this is severely limited by the lack of reliable data, even though such data have been accumulated over many years.

Gould and Roberts[2] give some of their results in a most useful form. They give the ratio of pulse breakdown fields to pressure as a function of the product of pressure and pulse length, indicating that this curve is then independent of frequency or pulse length. Data obtained from other investigators and made with different pulse lengths fit their curve very well. Figures 4 and 5 are an adaptation of their curve, extended to smaller values of $p\tau$ by use of MacDonald's data. Instead of using pulse breakdown fields as an ordinate the ratio of pulse breakdown fields to CW breakdown fields is used, expressed in decibels. MacDonald's data are for 100 mc with pulse lengths of 1.0 and 5.0 microseconds. His data for a frequency of 3 Gc and pulse lengths of 0.05 and 0.1 microseconds also fit the curve quite well. These curves may be used to find pulse breakdown fields quickly when pressure and pulse length are known. Use will be made of them later to find limiting fields for multifrequency breakdown.

The limitations of this theory on which the preceding discussions have been based are considered carefully by Brown and MacDonald.[14] MacDonald[4] points out that the limits of the theory are reached only in a few cases, and then only at such high altitudes that the curves are far from their minima and of little interest.

III. MULTIFREQUENCY BREAKDOWN

In the usual case of one antenna being used to transmit several signals simultaneously the signals will be separated by a frequency spacing small compared to any one of the signal frequencies. It may therefore be assumed that breakdown will occur at the same field strength for all signals. It will also be assumed for simplicity in the following work that all signals have the same amplitude and that this amplitude remains constant. This allows frequency modulation of the signals but not amplitude or pulse modulation.

Envelope Amplitude

When several signals of nearly the same frequency are added linearly as at an antenna, the slight difference in their frequencies is unimportant in determining breakdown. The factors of importance are their average frequency and the envelope of the wave which results from their addition.

Consider the phasor addition of three signals of frequencies ω_1, ω_2, ω_3.

Let median frequency $\omega_o = (\omega_1 + \omega_3)/2$. (19)

Let the middle frequency ω_2 differ from this mean by

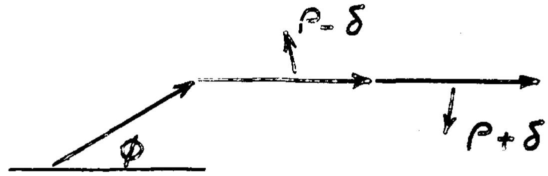
$$\delta = \omega_2 - \omega_0 \qquad (20)$$

Let

$$\rho = \omega_3 - \omega_0 = \omega_0 - \omega_1 \qquad (21)$$

Let ϕ be the angle of signal No. 2 at the time of direct addition of signals No. 1 and No. 3.

The phasors for the three signals may be drawn, holding the middle frequency phasor stationary



If each phasor has amplitude E_0 the resulting envelope for the magnitude of the three phasors is

$$E = \left\{ \left[\cos\phi + \cos(\rho - \delta)t + \cos(\rho+\delta)t \right]^2 + \left[\sin\phi + \sin(\rho - \delta)t + \sin(\rho+\delta)t \right]^2 \right\}^{1/2} E_0 \tag{22}$$

which reduces to

$$E = \left[1 + 4\cos(\delta t + \phi)\cos\rho t + 4\cos^2\rho t \right]^{1/2} E_o \tag{23}$$

The addition may also be performed graphically for particular choices of ρ, δ, and ϕ.

A computational program was carried out by digital computer to give the resultant envelope as a function of time for (a) $\delta = 0$, corresponding to three equally spaced frequencies, and ϕ a parameter, and (b) $\phi = 0$, corresponding to all signals adding directly at some instant, with δ a parameter.

Typical results of the computations are given by Figures 6 through 9. In Fig. 6, where the fields add directly at some instant, the overall envelope amplitude varies from $3\ E_o$ to 0. In Fig. 7, where one signal is at an angle of $\pi/4$ when the other two add directly the maximum and minimum values are $2.8\ E_o$ and $0.7\ E_o$.

In general,

$$E\ max = \left[5 + 4 \cos \phi\right]^{1/2} E_o \tag{24}$$

$$E\ min = \sin \phi\ E_o \tag{25}$$

Both Figs. 6 and 7 are for three equally spaced frequencies.

Figure 8 shows one case for unevenly spaced signals. For this figure, $\delta/\rho = 1/2$, which might, for example, correspond to three signals of 236, 238, and 244 mc. For Fig. 8, $\phi = o$, corresponding to all three signals adding directly to give a maximum of $3\ E_o$ at one instant of time.

Figure 9 is drawn for $\delta/\rho = 0.8$, which might correspond to three signals of 236,236.8, and 244 mc. All figures are plotted in terms of the difference frequency ρ. A comparison of the figures shows that a period of the envelope differs significantly from one curve to another. For $\delta/\rho = 0$ the envelope repeats each 2π radians in ρt. For $\delta/\rho = 1/2$ this repetition occurs each 4π radians; while for $\delta/\rho = 0.8$ the period is 10π radians. In general for

$$K = \rho/\delta \tag{26}$$

an integer the period is $2K\pi$ radians in ρt. For K not an integer, the period may be quite long. One case which may be easily calculated is for

$$K = \rho/\delta = 1 + 1/N \tag{27}$$

with N an integer. It may be shown that a period in ρt requires $2\ (N + 1)\ \pi$ radians.

In terms of the time T required for a period, assume that $\rho/2\pi =$ 4 mc. Then for the three cases discussed here we might have

δ/ρ	Frequencies (Mc)	T (microseconds)
0	236,240,244	0.25
0.5	236,238,244	0.5
0.8	236,236.8,244	1.25

Table I

It should be noted that in the calculations only ρ and δ are significant. The use of a median frequency of 240 mc is illustrative only. It would be easy to assume a case (for example by making N large in Eq. (27) with an extremely long period for the envelope.

The above discussion has been written for phase-coherent signals with no modulation. If the signals are derived independently and drift even slightly in frequency or if they are modulated in frequency the envelopes pictured here may be only approximations. In fact, the resultant envelopes will in general be aperiodic. It is, however, assumed in this work that breakdown power for phase-coherent signals will not differ appreciably from that for non-coherent signals of the same frequencies. This assumption is justified to some degree by measurements to be described in a later section.

The computations for four signal frequencies were not as extensively carried out as for the three frequencies because the additional variable would make a complete set of results extremely long. However, Fig. 10 shows the case of four evenly-spaced (in frequency) signals, all of which are so phased that they add directly at some instant. The amplitude here varies from $4\ E_o$ to 0. Figure 11 is the envelope of four evenly spaced signals with three adding directly and the fourth adding at an angle of $\pi/2$, at one instant of time. Here the peak is lowered and the minimum is raised, as compared to the first case.

Limits on Multifrequency Breakdown

Theoretically, the breakdown fields for any combination of signal frequencies may be obtained by considering a period of the wave, as shown in Figs. 6 through 11, to constitute a pulse of frequency equal to the median frequency of the applied signals. This would be done, as discussed in a previous section, by determining the net ionization constant ν as a function of the envelope of the applied electric fields. Integration of the appropriate equations, Eq. 16 or 17, would follow. In practice, data are lacking to allow this program to be carried out; in particular, data are not available for the small amplitude portions of Figs. 6 through 11. In addition, since these "pulses" are repetitive it would not be sufficient to integrate over one period alone. This would further complicate the situation since for integration over a long period of time the more general Eq. (17), including diffusion effects, might be necessary.

These two problems, especially the lack of data, make direct integration from a knowledge of the field envelope an unrealistic solution. Therefore, simpler methods have been used to find limits on the breakdown fields.

Minimum Fields

The minimum power that can be radiated before an antenna breaks down will occur if the peaks of the envelope determine breakdown by acting as a CW breakdown field. For phase-coherent addition with n signals adding in phase at some instant

$$E_{max} = nE_o = E_{cw} \tag{28}$$

where E_{cw} is the cw field required for breakdown.

$$E_o = E_{cw}/n \tag{29}$$

The powers are related by

$$P_o = P_{cw}/n^2 \tag{30}$$

where P_o is the power of each signal. Total power for n signals is

$$P = nP_o = P_{cw}/n \tag{31}$$

or

$$P/P_{c\omega} = 1/n \tag{32}$$

Equation (32) gives the ratio of minimum n-frequency power which can be radiated before breakdown to the single frequency power causing breakdown of the same antenna. Table II gives this ratio in decibels for various numbers of signals.

n	P/P_{cw} (db)
1	0
2	-3.02
3	-4.78
4	-6.03
5	-7.00
6	-7.78

Table II

Minimum Multifrequency Power

Table II is applicable to phase-coherent signals which add in phase at some instant. Non-coherent signals will also add directly at some instant; therefore, Table II applies to the non-coherent case also. This is true no matter what the individual signal frequencies are.

For three coherent signals which add with a different phase angle, the maximum field is

$$E\ max = \left[5 + 4\cos\phi\right]^{1/2} E_o \qquad (24)$$

Following the development of Eqs. (28) through (32) gives the minimum three-frequency power that can be radiated before breakdown,

$$P\ min/P_{cw} = \frac{3}{5 + 4\cos\phi} \qquad (33)$$

This ratio is shown as a function of angle ϕ in Fig. 12. Maximum limits and measured data are also shown in Fig. 12. These will be discussed in later sections.

Maximum Fields (A) - Envelope Valleys

The maximum power that can be radiated before breakdown will occur if the valleys of the envelope determine breakdown by acting as a CW field. For three equally spaced coherent signals, the minimum field is

$$E\ min = E_o \sin\phi \qquad (25)$$

It follows that the maximum power that can be radiated before breakdown, compared to CW breakdown, is

$$P\ max/P_{cw} = \frac{3}{\sin^2\phi} \qquad (34)$$

This ratio is shown in Fig. 12.

For three coherent signals that are not equally spaced in frequency the minimum fields (envelope valleys) may also be determined and thus the maximum power which can be radiated before breakdown. Figure 13 shows the maximum power limit as a function of $^1/_K = \delta/\rho$. Since the smallest envelope valley can only be determined by computing the entire envelope the maximum power limits are given only for discrete values of K. Also shown in Fig. 13 is a line representing the minimum power that can be radiated, assuming that all three signals add directly at some instant. Figure 13 includes a curve of measured breakdown data. This will be discussed later.

For non-coherent signals and for coherent signals with particular phase relations, the smallest envelope valleys may be zero. This would give a maximum limit of infinite power radiated before breakdown. This is, of course, neither correct nor useful.

Maximum Fields (B) - Single Pulse Breakdown

Another limit on the maximum power radiated before breakdown is the breakdown caused by a portion of the envelope, Figs. 6 through 11, for example. As discussed before, the full envelope cannot be integrated to find breakdown, for practical reasons. Consider, however, that the largest (approximately sinusoidal) pulse of the envelope is considered to cause breakdown, without assistance from any other portion of the envelope. The limit obtained by this assumption may be much larger than the values obtained by measurements, but it has some potential usefulness.

Examination of all the envelopes computed for the various values of K showed the largest pulse of all envelopes to have the same height ($3\ E_o$) and approximately the same width for constant ρ . Therefore, the calculations to be discussed here were carried out for the case of three equally spaced signals, $K = \infty$.

The determination of breakdown was carried out in the following manner: The equation of the envelope, Eq. (23) was used, with $\phi = 0$ and $\delta = 0$, giving

$$E = \left[1 + 4 \cos \rho t + 4 \cos^2 \rho t\right]^{1/2} E_o \tag{35}$$

An arbitrary portion of the pulse was chosen (at the center of the pulse) and the height, h, at that point calculated from Eq. (35). The assumption was made that the pulse causing breakdown would then have the height h and the arbitrary width τ as shown in Fig. 14. Thus for purposes of calculating breakdown the approximately sinusoidal pulse was replaced by the rectangular pulse. Next, Figs. 4 and 5 were used to give the rectangular pulse breakdown fields in terms of CW breakdown fields.

For breakdown to occur, value h must be equal to Ep, the pulse breakdown field of Figs. 4 and 5. Then

$$\frac{h}{E_o} = \frac{Ep}{E_o} \tag{36}$$

and

$$\frac{E_o}{E_{cw}} = \frac{E_o}{Ep}\,\frac{Ep}{E_{cw}} = \frac{E_o}{h}\,\frac{Ep}{E_{cw}} \tag{37}$$

and

$$\left.\frac{E_o}{E_{cw}}\right]_{db} = \left.\frac{Ep}{E_{cw}}\right]_{db} - \left.\frac{h}{E_o}\right]_{db} \tag{38}$$

Figures 4 and 5 give the first term of Eq. (38) directly, where E_p is the pulse breakdown field and E_{cw} is the CW breakdown field.

Since breakdown is a function of pulse width and height, several combinations of width and height were tried. The one giving a minimum value of E_o / E_{cw} was assumed to be the one at which breakdown actually occurred.

The procedure is demonstrated by Table III, for a pressure of 0.2 mm Hg and a difference frequency for the three equally spaced signals of $\rho/2\pi = 4$ mc.

τ (sec)	$p\tau$	h/E_o (Eq. 35)	E_p/E_{cw} (Fig. 4)	E_o/E_{cw} (Eq. 38)
2.5×10^{-8}	5.0×10^{-9}	9.26 db	18.7 db	9.44 db
3.75×10^{-8}	7.5×10^{-9}	8.92	17.8	8.88 *
4.5×10^{-8}	9.0×10^{-9}	8.60	17.5	8.90 *
5.0×10^{-8}	10^{-8}	8.38	17.3	8.92 *
5.5×10^{-8}	1.1×10^{-8}	8.12	17.2	9.08
6.25×10^{-8}	1.25×10^{-8}	7.66	17.0	9.34
10^{-7}	2.0×10^{-8}	6.76	16.2	9.44

* Breakdown at this value

Table III

Total power required for breakdown can then be calculated from

$$P/P_{cw} = 3\,(E_o/E_{cw})^2 \qquad (39)$$

or

$$P/P_{cw}\Big]_{db} = \frac{E_o}{E_{cw}}\Big]_{db} + 10 \log 3 \qquad (40)$$

giving, for the case above,

$$P/P_{cw}\Big]_{db} = 8.9 + 4.78 = 13.7 \text{ db}$$

Table IV gives these maximum single pulse breakdown fields for difference frequencies of three signals ranging from 0 to 10 mc.

Difference frequency $\rho/2\pi$, mc	P/P_{cw} (db)
0	-4.78
.01	2.4 *
.05	6.8
.1	7.9

Table IV Continued

Difference frequency $\rho/2\pi$, mc	P/P_{cw} (db)
.2	9.0
.4	10.0
.6	10.7
.8	11.2
1	11.5
2	12.6
4	13.7
6	14.5
8	15.1
10	15.7

* Macdonald's 5 microsecond data used. Use of his 1 microsecond data would add about 2 1/2 db to this value. Macdonald's 1 microsecond data used for all other calculations.

Table IV

It should be noted that the maximum breakdown limit determined in this fashion is a function of pressure. It also requires the use of data obtained by other investigators. The other limits used do not depend either on pressure or on measured data.

IV. BREAKDOWN MEASUREMENTS

Measuring System and Procedures

The measuring setup shown in Fig. 15 was used to measure power required for antenna breakdown for single frequencies and for multiple frequencies, coherent and non-coherent.

The oscillators are General Radio Unit oscillators operating in the range 236 mc to 244 mc. The GR Bridge oscillator, when used, operates at 4 mc. The first amplifiers are Radiation, Inc. Types 3021 and 3115 oscillators modified by removing their crystals and applying the signal to be amplified to the control grid of the second mixer stage. The final amplifier is a Tele-Dynamics, Inc. RF power amplifier, Type 1101. The multiplexers used for separating the components of the modulated signal for separate amplification (first multiplexer) and recombining the three signals (second multiplexer) are Rantec Type MT - 433.

Three phase-coherent signals were obtained by modulating a 240 mc signal by a 4 mc modulating signal in the crystal-diode modulator. The phase of one signal was adjusted by inserting sections of air line before one amplifier. The non-coherent signals were obtained by feeding the three oscillator signals directly to the first amplifier, bypassing the first multiplexer.

The antenna is a stub consisting of an inner conductor of diameter 1/8 " extending vertically out of a hollow sleeve of inner and outer diameters respectively 9/32 " and 5/16 " . The distance from ground plane to the top of this vertical outer sleeve is 4" . The inner conductor, and antenna proper, protrudes 9 1/8" from the sleeve. Both the inner conductor and outer sleeve are soldered to appropriate components of a type N connector and then connected in the usual manner to a type N connector extending through the ground plane (the floor of the vacuum chamber). One foamed polystyrene spacer is used between the antenna and outer sleeve.

Measured antenna impedances at three frequencies used (measured not at the inaccessible antenna terminals but at a connector on the front of the vacuum chamber perhaps 8 inches from the antenna connector) are shown in Table V.

Frequency, Mc	VSWR, db	Impedance
236	4.5	50 + j24
240	4.0	50 + j27
244	4.4	48 + j27

Table V

As a check on the measured impedances, power required for breakdown was also measured as a function of frequency. This has a much greater validity than the impedance measurements, since the measuring apparatus is in the near field of the antenna, and is not the same for impedance measurements as for breakdown measurements. Table VI shows the single-frequency breakdown power as a function of frequency, related to breakdown power at 236 Mc.

Frequency, Mc	Breakdown Power, db
236	0
237	-0.07
238	-0.25
239	-0.34
240	-0.68
242	-0.61

Table VI

Table VI shows that the antenna, as used for multifrequency breakdown experiments, has a sufficient bandwidth. This information is shown also by Fig. 16.

The pressure used in the vacuum chamber was about 0.2 mm Hg, corresponding to an altitude of about 200,000 ft.[4] Each type of measurement was repeated several times as the pressure slowly increased because of small air leaks. These measurements were then averaged to give one data point. The same small range of pressures was used for each data point. No cold traps were provided, and normal room air was used with no attempt to remove moisture. Temperature was also left free to vary over a comfortable room temperature range.

The pressure used allowed very nearly a minimum breakdown power, important because of the relatively low power capability of the system. In addition, this pressure caused the electron loss to be controlled by attachment at both CW and with multiple frequency signals. As mentioned previously, pulse breakdown is apt to be attachment controlled even if CW breakdown with the same antenna is controlled by diffusion. The choice of antenna and pressure used in this experimental work insured the same breakdown mechanism for all types of breakdown, giving more meaning to the results.

Some investigators use an ionizing source in the vicinity of the antenna to assure a constant supply of electrons to initiate breakdown. One was not used in this study, but even so, constancy of breakdown power was relatively good.

Single-frequency breakdown was measured by slowly increasing the plate supply voltage to the first amplifier while observing the readings of the two power meters. Breakdown was assumed to occur with the appearance of a visible glow on approximately the upper 1" of the antenna. Simultaneously the reflected power increased very sharply. The power readings given in this report are the values just before breakdown. An interval of about one minute was allowed between breakdowns, since it was found that in the first few seconds after one breakdown another could be initiated with lessened power. After pumping down the vacuum chamber the first breakdown power level was not recorded since it tended to be quite erratic. No distinction could be made in the measurements between the appearance of the visible glow and the increase in reflected power, so to the limits of discrimination available these were considered to occur simultaneously.

When breakdown occurred, the power read by the forward-power meter remained relatively constant, but the reflected power increased greatly. Overall, the radiated power dropped by about 2 db. As stated, the visible glow appeared only near the antenna tip, leaving a large part of its length free for radiation. It is probable that for other antennas, such as slots for example, the radiated power on breakdown would decrease much more than the value obtained here.

Multifrequency breakdown was measured by replacing the forward-power meter by the spectrum analyzer so that the amplitude of each signal could be monitored simultaneously. The plate supply voltage to each amplifier was increased slightly in turn, keeping the signal amplitudes as displayed by the spectrum analyzer approximately constant, until breakdown occurred. The power meter then replaced the spectrum analyzer. The final amplifier was switched off to extinguish the glow, and then switched on and the power meters read before the glow reappeared.

In practice the measurements were not always made independently. To minimize the effects of ambient temperature, air composition, etc. a multifrequency breakdown measurement was alternated with a single frequency measurement when it was desired to compare the two. As implied, however, several measurements of each were averaged before comparison.

Power Required for Breakdown

Power required to break down the antenna at 236 Mc was in the neighborhood of 7 watts. Other antennas were tested which required less power, but no sufficiently broad-band antenna was found. Minimum average power for one group of readings was 6.3 watts; maximum average power for another group, taken on another day, was 7.9 watts, a difference of 1.0 db. A similar variation was found for multifrequency powers. As stated, this variation was largely overcome by comparing multifrequency power to single frequency power for alternated measurements. This variation is for average values of groups as compared to each other. For the several readings in each group, as pressure varied over a small range, most of the individual readings were found to lie within 0.1 db of the group average for single-frequency data and within 0.2 db for multifrequency data. This gives confidence that the measurements do provide meaningful data.

Two - Frequency Breakdown

Breakdown power was measured for two independent signals as a function of their difference frequency. This was done for combinations of 236 and 238, 236 and 240, 236 and 242, and 236 and 244 Mc. Figure 17 shows the results. The total power in the two signals required for breakdown was about 4 1/2 watts, as compared to about 6 1/2 watts for single frequency (236 Mc) breakdown. This difference seemed to be largely independent of the frequency difference.

Three - Frequency Breakdown, Coherent Signals

Three phase-coherent signals of frequencies 236, 240, and 244 Mc were used to measure breakdown as a function of ϕ , the phase angle of the 240 Mc signal when the other two signals add at 0^{o}. The result is shown in Fig. 12, which also shows the minimum and maximum theoretical limits discussed in a previous section. The measured curve appears to follow the theoretical minimum, differing from it by about 3 db. This experiment does not, of course, duplicate conditions existing for modulated and incoherent signals.

Three - Frequency Breakdown as a Function of Difference Frequency

Three evenly spaced, but independently derived and therefore incoherent, signals were measured. Results are shown by Fig. 18. The lowest frequency in all cases was 236 Mc and the difference frequency ranged from 1 to 4 Mc. Fig. 18 shows that slightly more power can be transmitted before breakdown for the greater difference frequencies. It is to be noted that changing the frequency spacing did not change the breakdown power appreciably for two signals, Fig. 17.

Three - Frequency Breakdown, Unequal Frequency Spacing

Figure 13 shows the results of measurements made by holding the two end frequencies constant at 236 and 244 Mc while changing the frequency of the remaining signal. The graph is given in terms of $1/k = \delta/\rho$, where $\rho/2\pi$ is now 4 Mc and $\delta/2\pi$ is the difference in frequency of the middle-frequency signal from 240 Mc. Breakdown power is seen to remain relatively constant, regardless of the frequency of the middle-frequency signal. The value shown in Fig. 13 agrees also with that shown in Fig. 18 for a difference frequency of 4 Mc. It seems that for three signals, of equal amplitude, breakdown is determined by the difference between the two extreme frequencies and allows slightly greater power to be transmitted if this difference is larger.

Four - Frequency Breakdown

No measurements were made to determine breakdown with the radiation of four signals, because of the tedious and lengthy methods of measurement used. However, it was noted from the two- and three-frequency breakdown measurements that the actual breakdown power was about 1.5 db (2 signals) to 3 db (3 signals) from the theoretical minimum power. Figure 19 has a speculative interest only, but nevertheless it may be worth some consideration. It shows, as a function of the number of equal amplitude signals, measured breakdown power as compared to single-frequency breakdown power. It also shows the theoretical minimum power for each number of signals. Note that the measured value for three signals lies in a range depending on frequency separation. The points are connected by smooth curves. The measured curves are extended on to 4 signals, even though no measurements were made. It is intuitively felt that this procedure has merit, since it is unlikely that breakdown by 4 signals will differ appreciably in character from that of 2 and 3 signals. The breakdown for 4 signals is expected to have a range of values, like that shown, depending on frequency spacing. In practice, some leeway must certainly be allowed for the possible inaccuracy of this extrapolation, and it is likely that the smallest value of about -3 db would be observed.

Multifrequency Breakdown, Unequal Amplitude Signals

Again no measurements were made because of the many possible combinations of signal frequencies and amplitudes. As with equal amplitude signals a theoretical minimum total power can be obtained by assuming that the signals, adding directly, act to cause CW breakdown. Thus for m signals of unequal amplitude, with respective powers $P_1, P_2, \ldots P_m$, the total radiated power is

$$\sum_{n=1}^{m} P_n$$

Setting the square of the sum of the amplitudes equal to the power for CW breakdown at the same mean frequency and altitude gives the following criterion as a lower limit on the power which can be radiated

$$\left[\sum_{n=1}^{m} \sqrt{P_n}\right]^2 = P_{cw} \tag{41}$$

The measurements described previously showed that for equal amplitude signals the left side of Eq. (41) could in practice exceed the right side by 1.5 db for two signals and 2.4 to 3.1 db for three signals before breakdown occurred. At the same time the measurements showed that

$$\sum_{n=1}^{m} P_n < P_{cw} \tag{42}$$

by 1.5 db for two signals and 1.7 to 2.4 db for three signals at the time of breakdown.

Consider two signals of unequal amplitude. An upper limit on the radiated power is that obtained by ignoring the smaller signal, making the signal power of the greater signal equal to CW breakdown. A lower limit could be obtained by assuming both signals to be of the same amplitude, that of the larger signal. Then the total power of the two signals must be at least 1.5 db below the CW breakdown power. Thus the breakdown for the two signals will be in the region from 0 to 1.5 db below the CW breakdown power for that antenna, frequency, and altitude.

With three signals unequal in amplitude, the breakdown value for total power will be in the range 0 to 2.4 db below CW breakdown. The total power breakdown value for four signals, assuming the extrapolation of a previous section to be valid, will be somewhere in the range 0 to 3 db below CW breakdown power.

V. CONCLUSIONS

It has been seen from the work reported herein that when an antenna is used to radiate more than one signal less total power can be radiated before breakdown than when one CW signal only is transmitted. Very little can be done to improve this situation. Use of a wide frequency separation for three transmitted signals appears to help slightly, but not significantly. The center frequency appears to matter little.

The problem is basically one of improving the breakdown characteristics for CW transmission, and although that is not the subject of this report the curves of CW breakdown at various frequencies show clearly that a considerable improvement can be expected by using the higher frequencies.

This report has given values of total power, in terms of CW breakdown power, which will cause breakdown for two, three, and, by extrapolation, four signals radiated simultaneously.

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VII. FIGURES

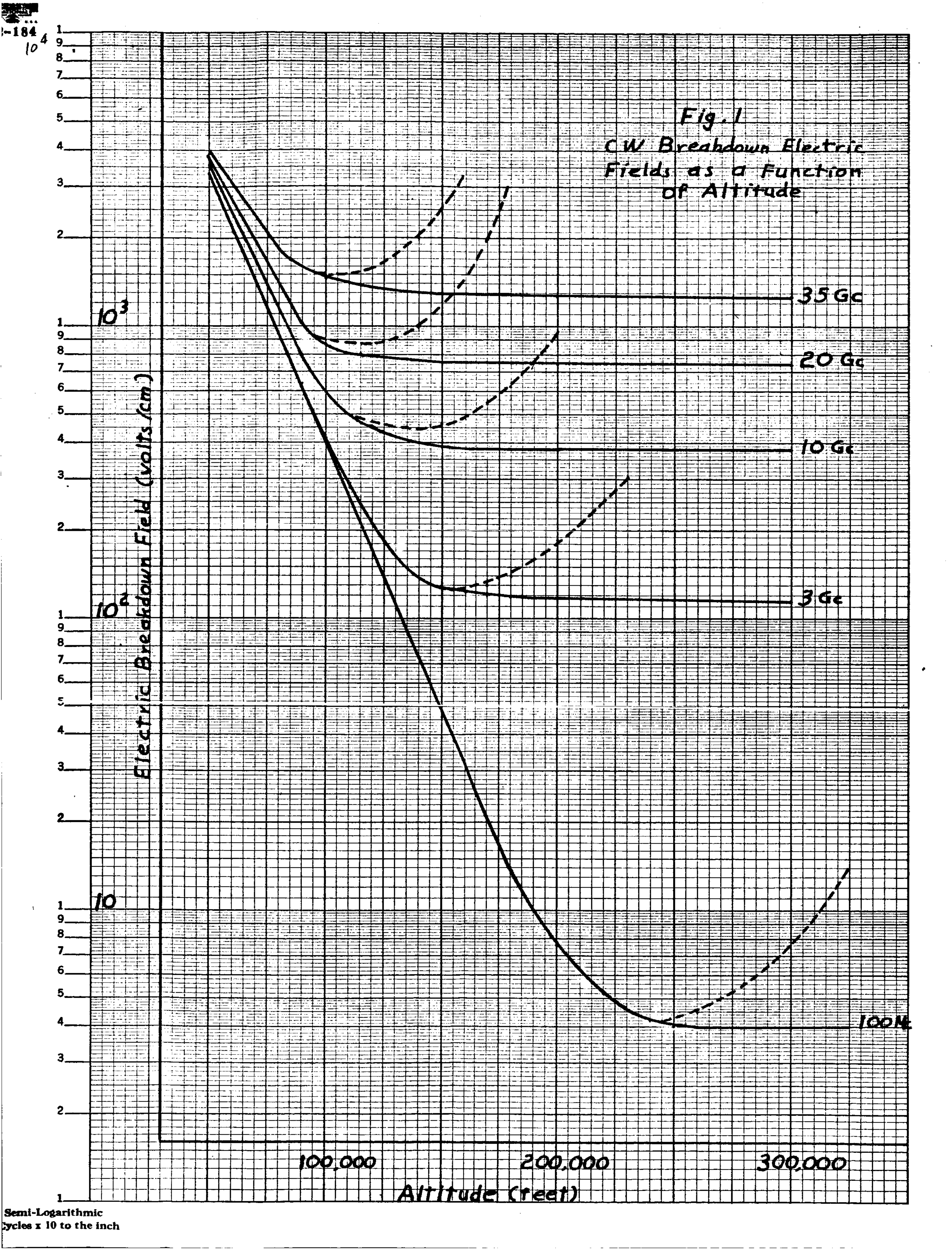
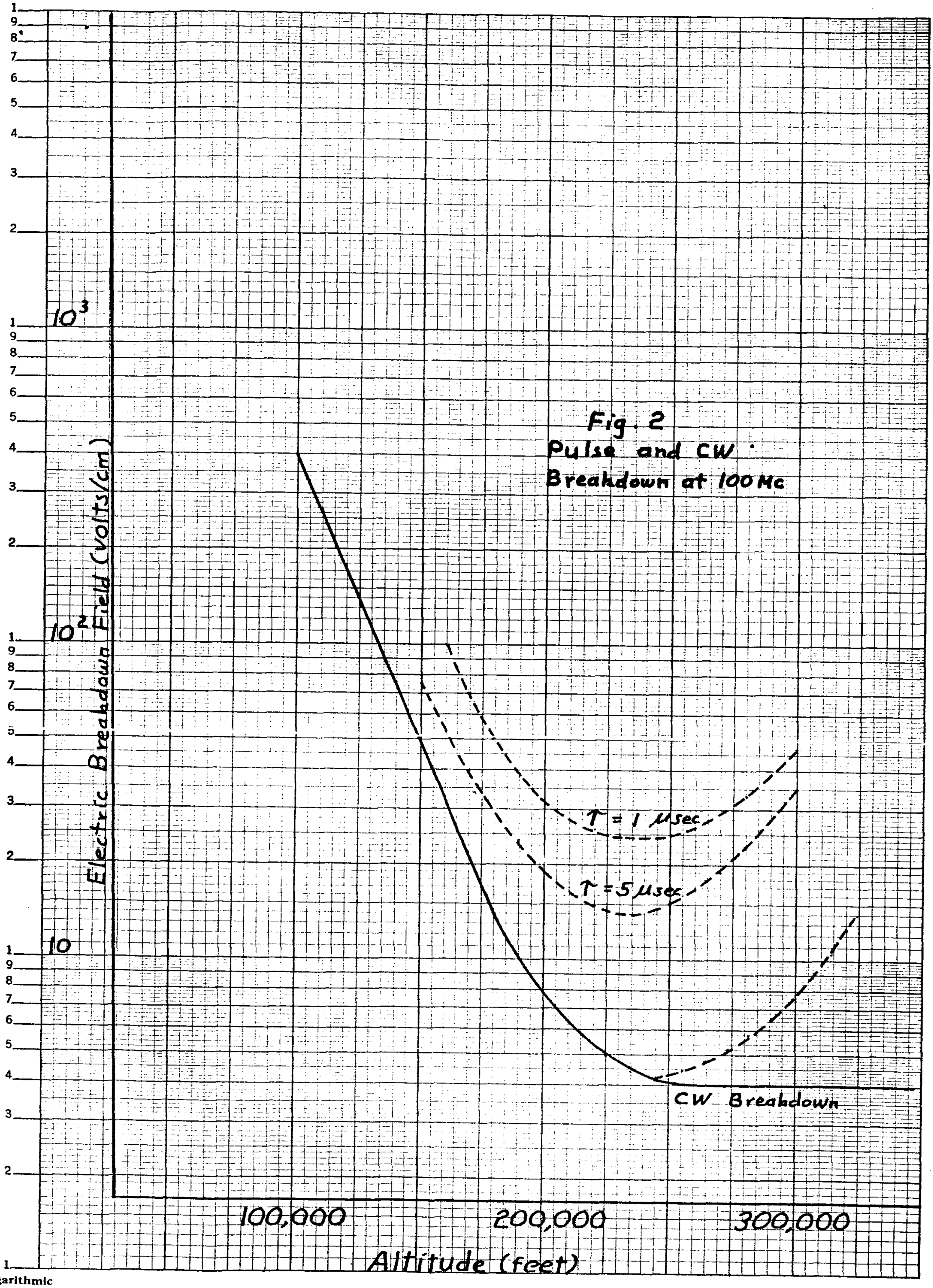


Fig. 1
CW Breakdown Electric Fields as a Function of Altitude
35 Gc
20 Gc
10 Gc
3 Gc
100Mc
Electric Breakdown Field (volts/cm)
Altitude (feet)
100,000
200,000
300,000
10^4
10^3
10^2
10
Semi-Logarithmic
Cycles x 10 to the inch



12-184
Semi-Logarithmic
ycles x 10 to the inch

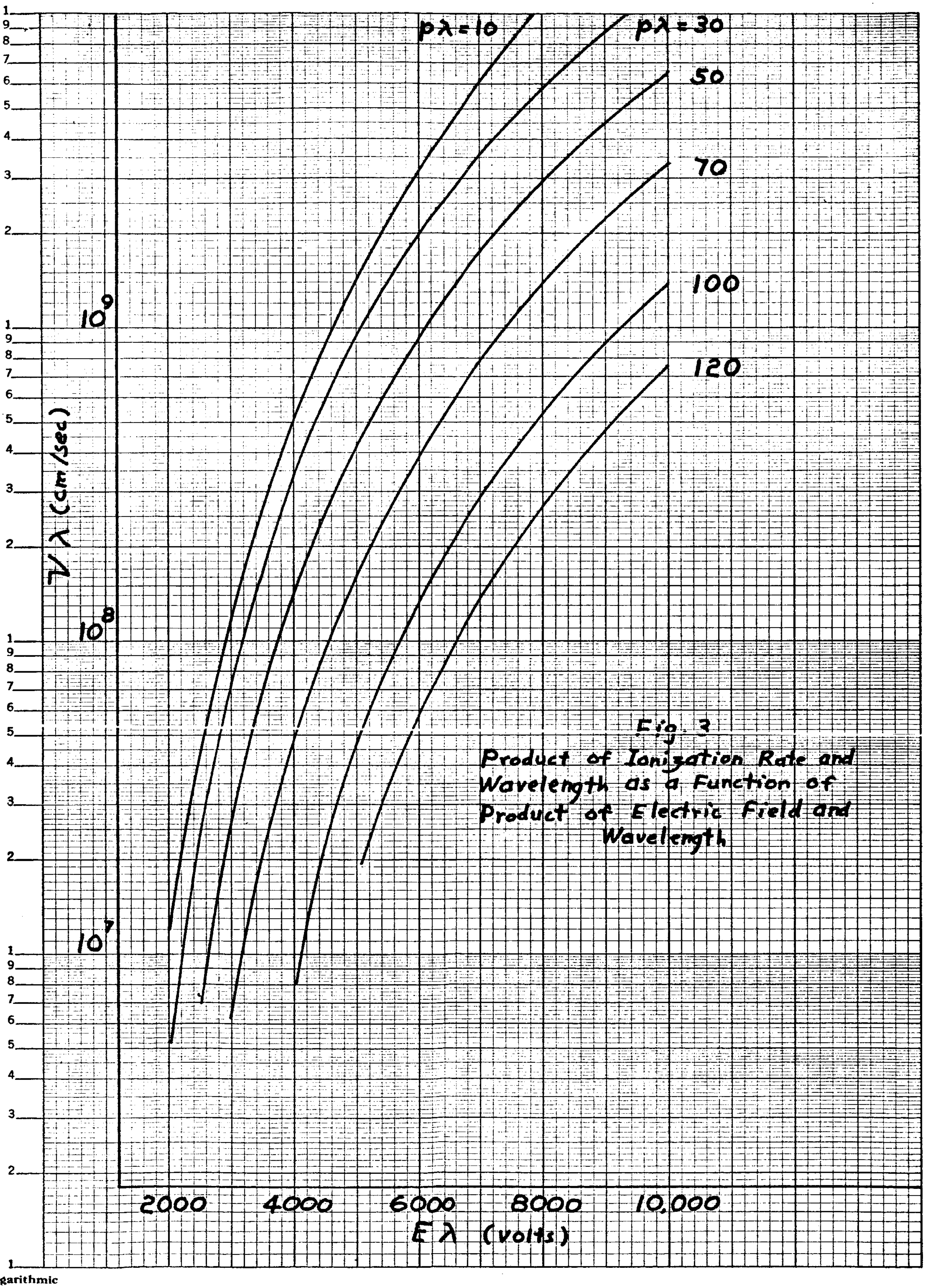
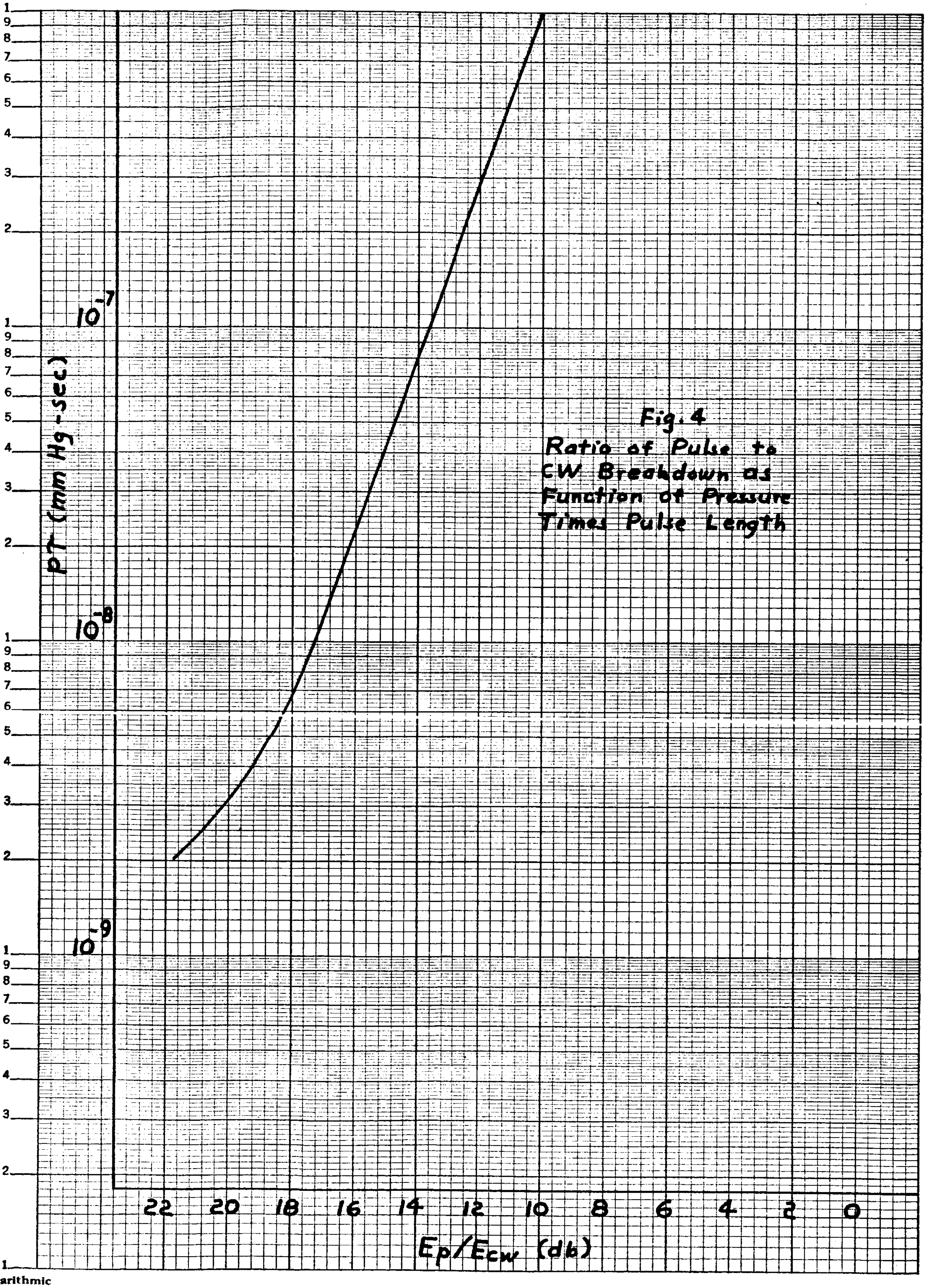


Fig. 3
Product of Ionization Rate and Wavelength as a Function of Product of Electric Field and Wavelength



Semi-Logarithmic

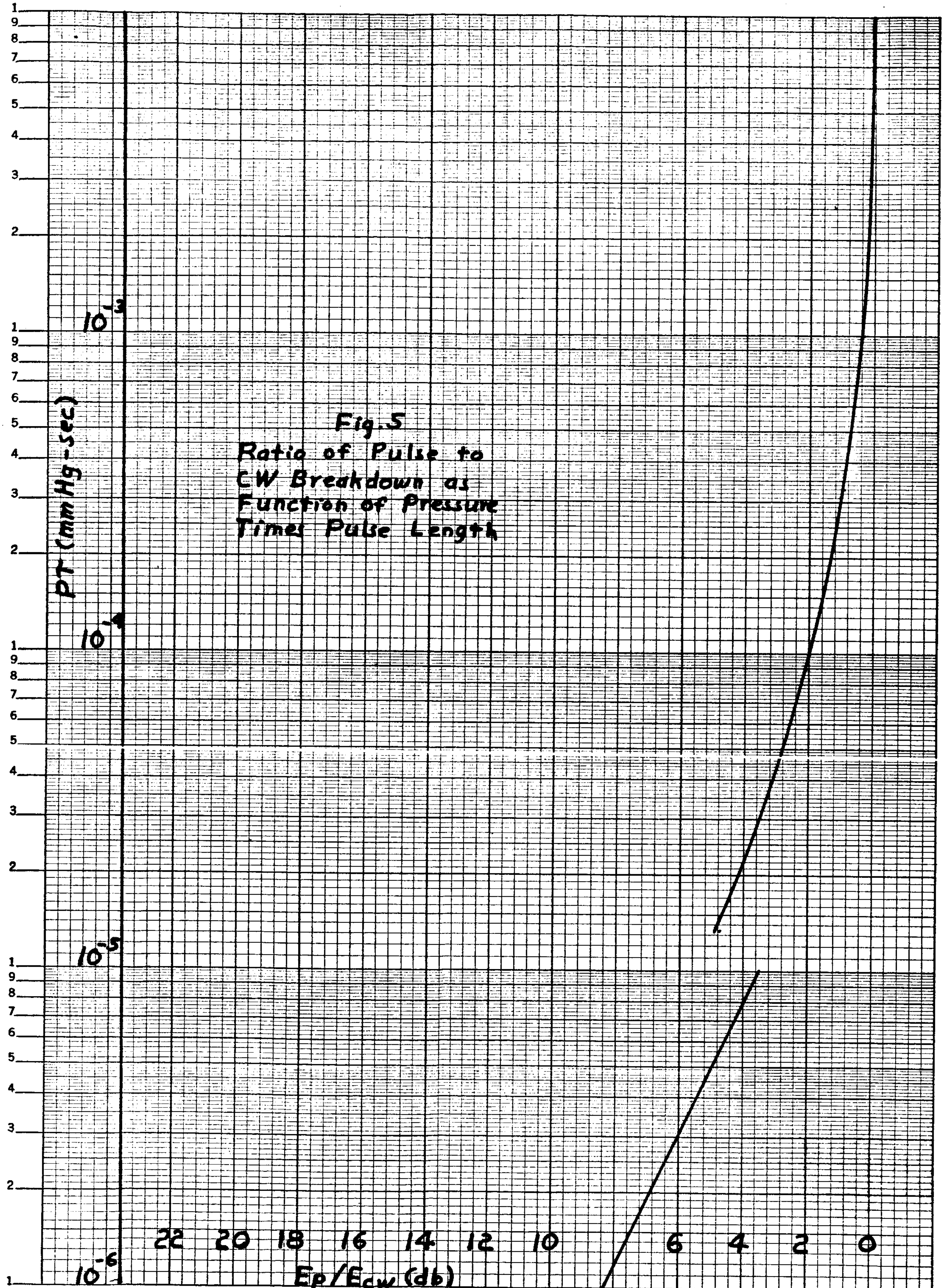


Fig. 5
Ratio of Pulse to CW Breakdown as Function of Pressure Times Pulse Length
PT (mm Hg-sec)
10^-3
10^-4
10^-5
10^-6
22 20 18 16 14 12 10 6 4 2 0
Ep/Ecw (db)

Fig. 6
Envelope of Electric
Fields for Three Signals,
Equal Spacing, $\delta = 0$, $\phi = 0$

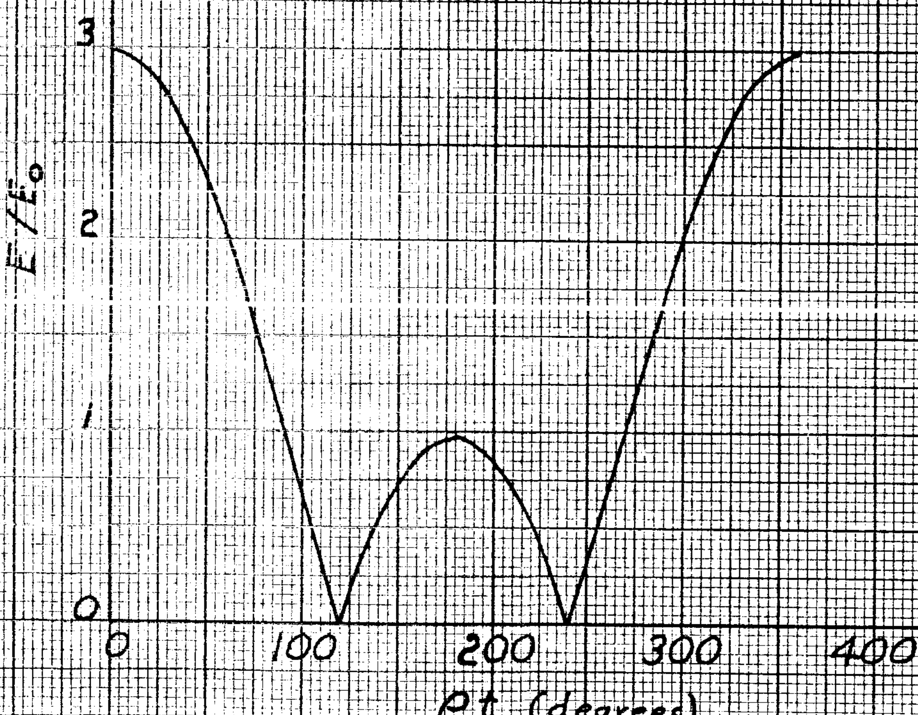


Fig. 7
Envelope of Electric
Fields for Three Signals,
Equal Spacing, $\delta = 0$, $\phi = \pi/4$

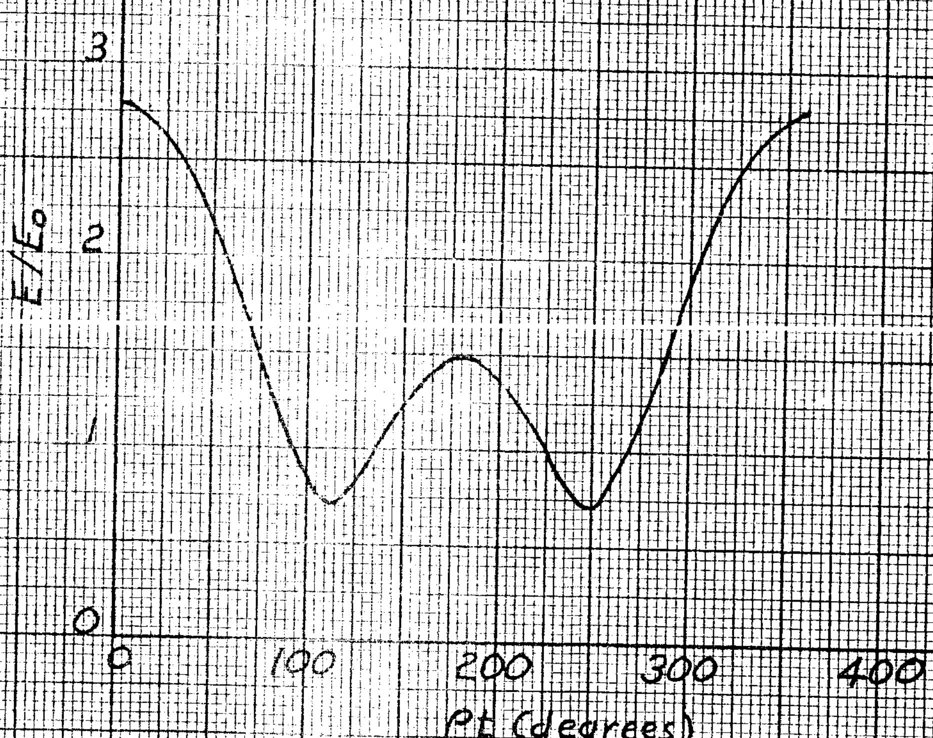
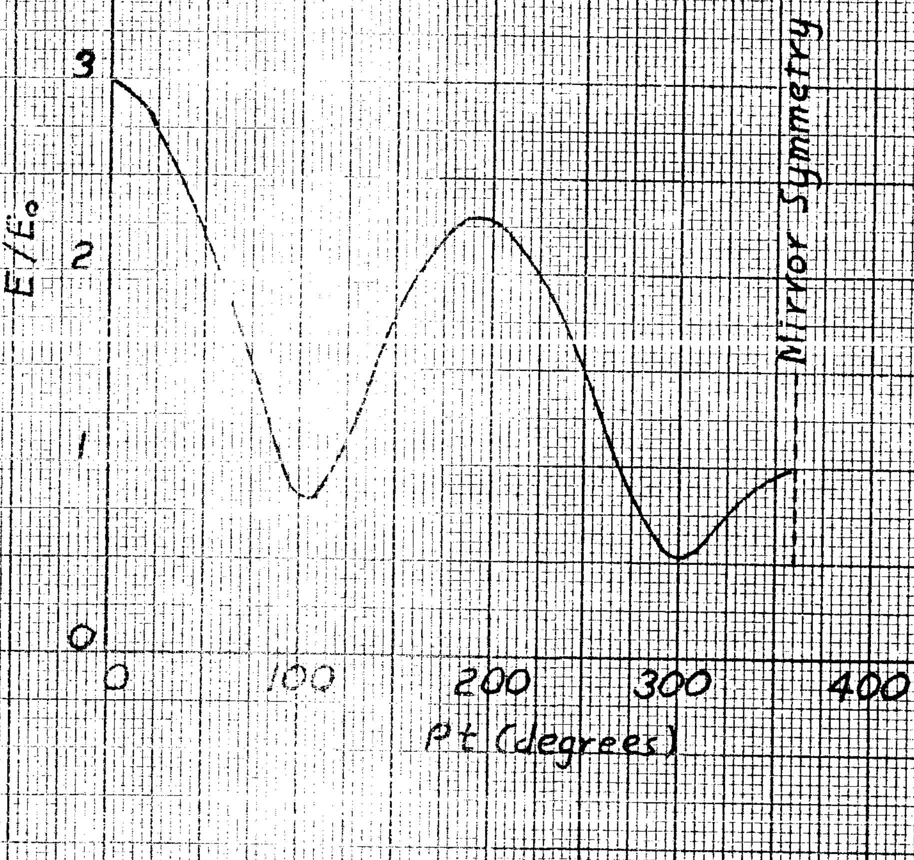


Fig. 8
Envelope of Electric
Fields for Three Signals
Unequal Spacing, $\delta/p = 1/2$
$\phi = 0$



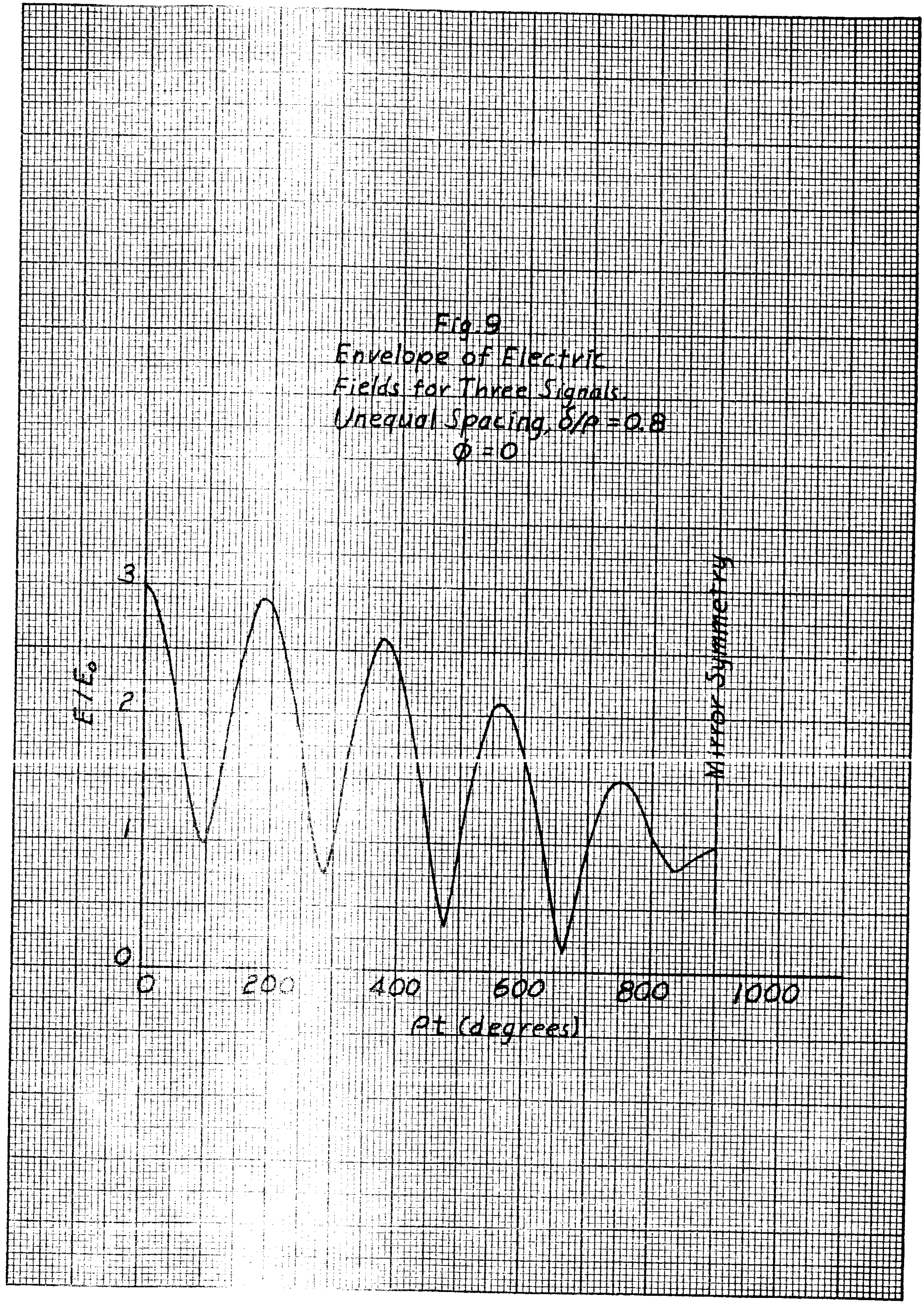
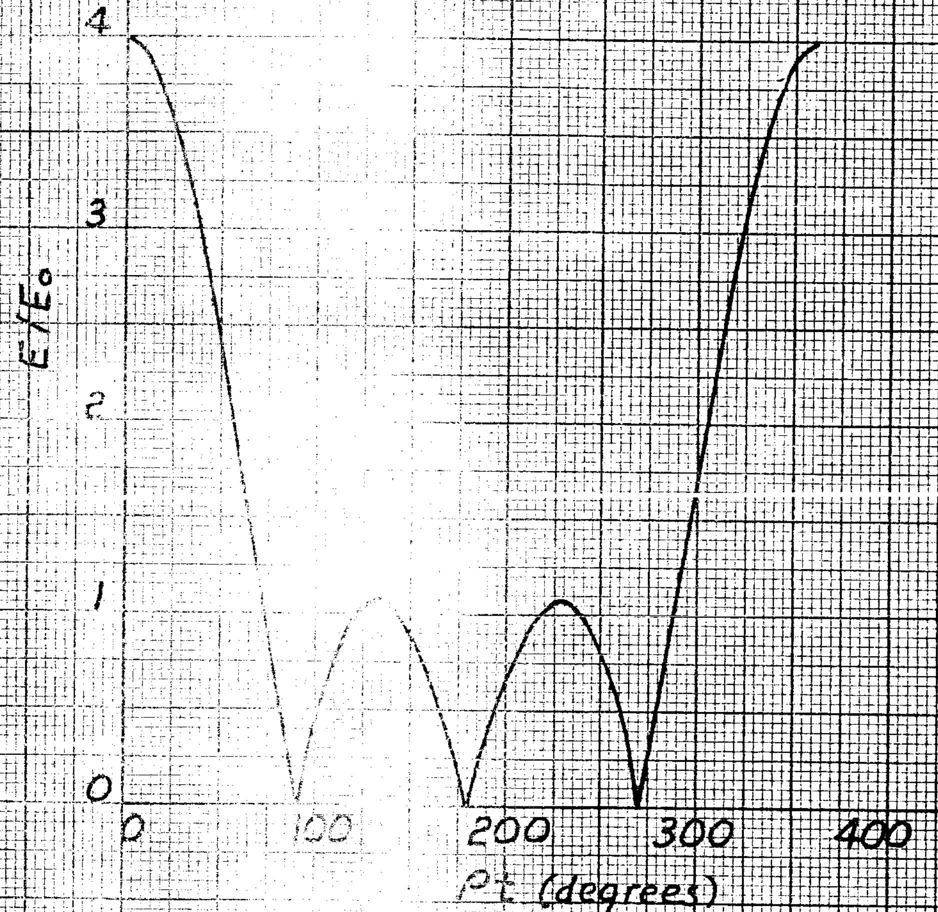


Fig. 9
Envelope of Electric
Fields for Three Signals.
Unequal Spacing, $\delta/P = 0.8$
$\phi = 0$

Fig. 10
Envelope of Electric
Fields for Four Signals.
Equal Frequency Spacing $P/2\pi$
Signals Add Directly
at $Pt = 0$.



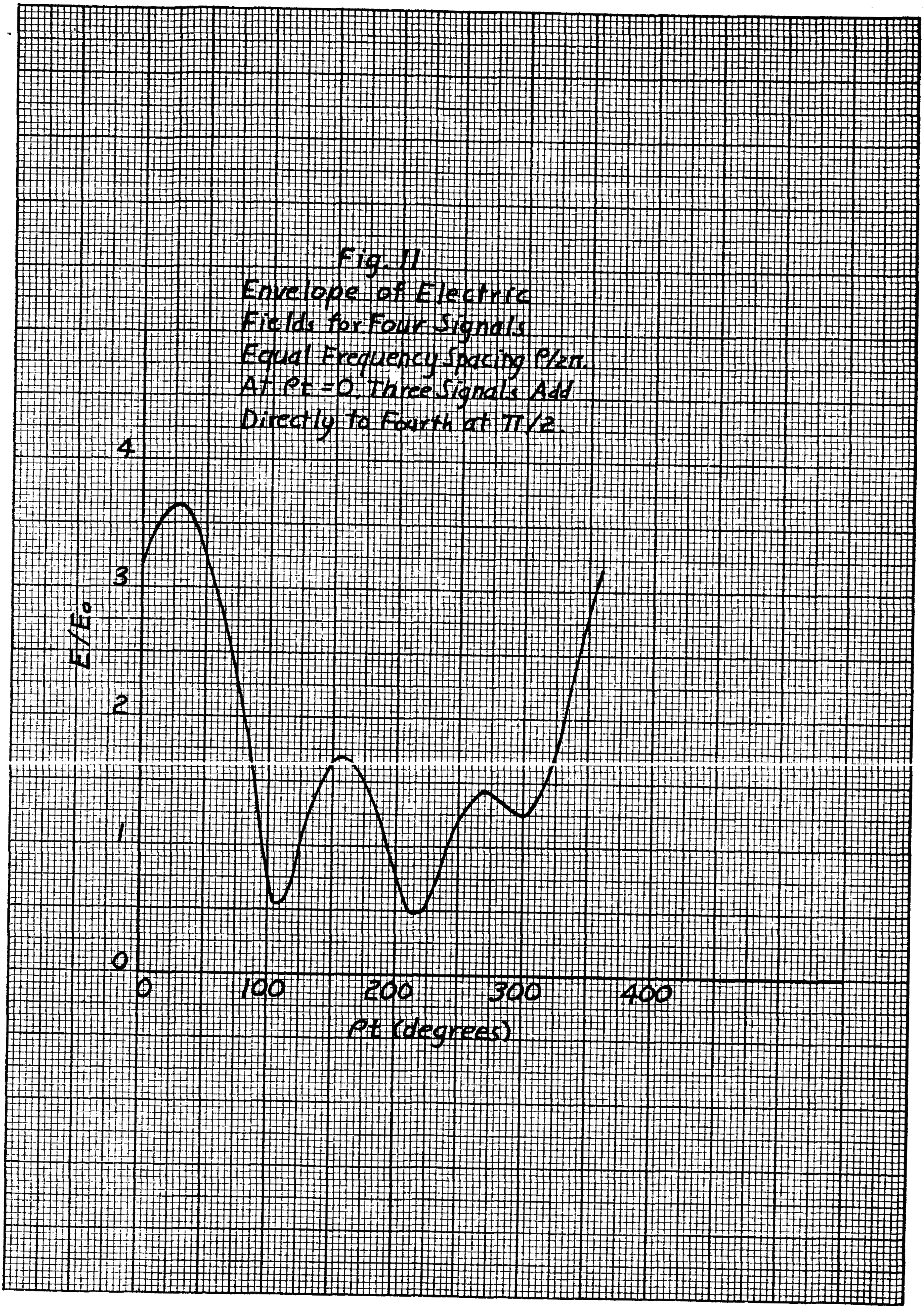


Fig. II
Envelope of Electric
Fields for Four Signals
Equal Frequency Spacing P/2π.
At Pt = 0. Three Signals Add
Directly to Fourth at π/2.
E/E₀
4
3
2
1
0
0
100
200
300
400
Pt (degrees)

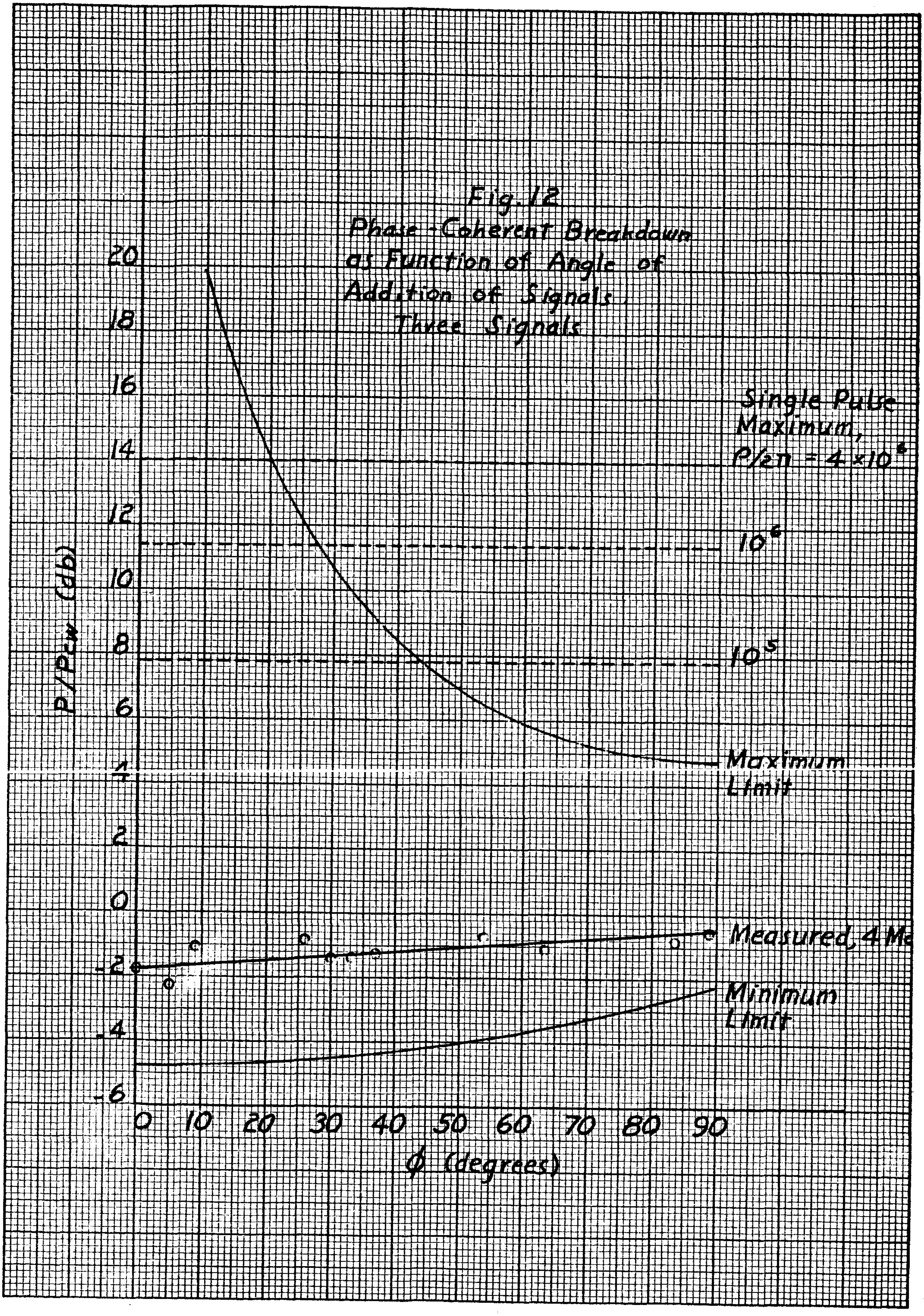


Fig. 12
Phase-Coherent Breakdown as Function of Angle of Addition of Signals
Three Signals
P/Pcw (db)
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
0
10
20
30
40
50
60
70
80
90
ϕ (degrees)
Single Pulse Maximum, P/2π = 4×10⁶
10⁶
10⁵
Maximum Limit
Measured, 4 Mc
Minimum Limit

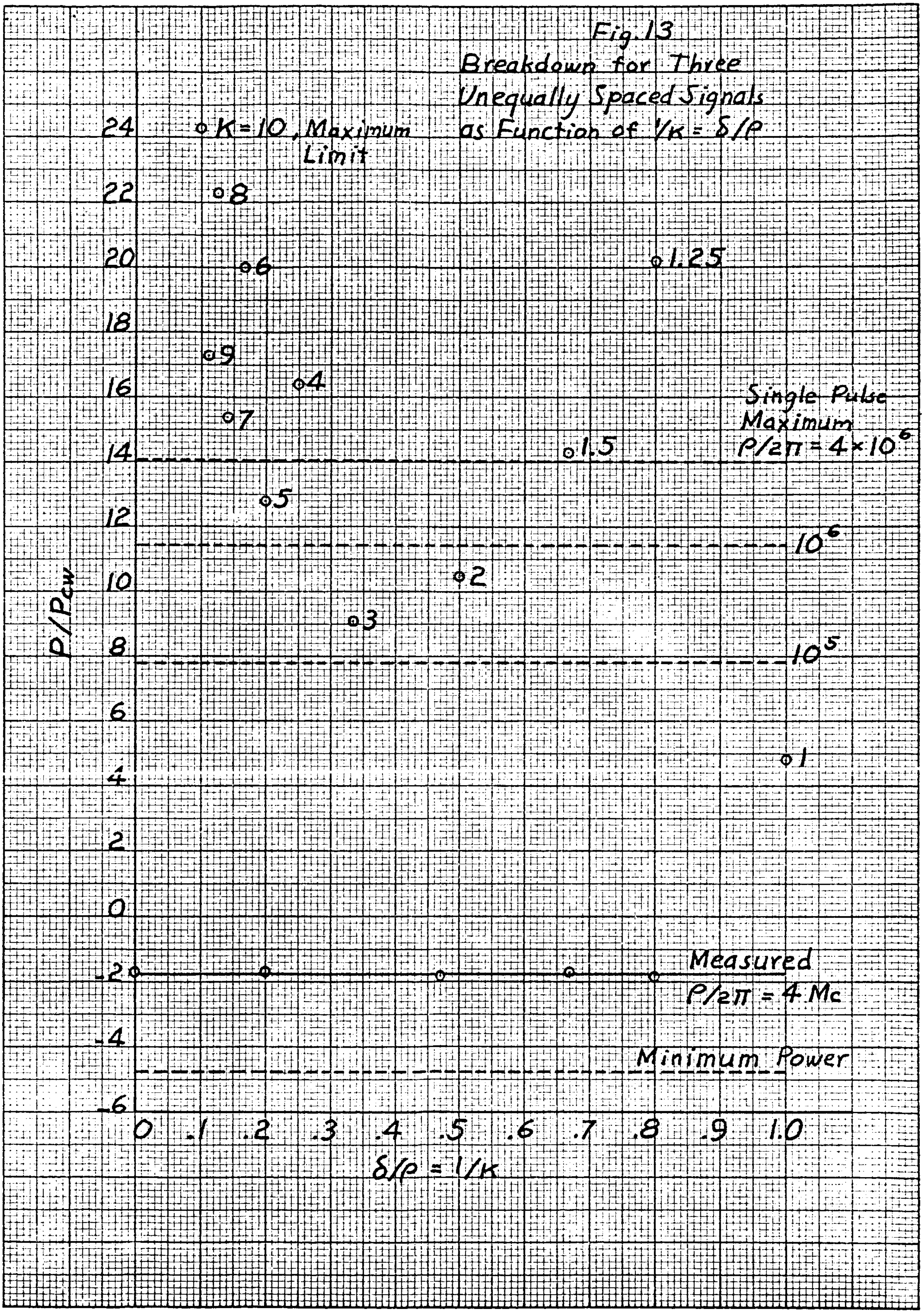


Fig. 13
Breakdown for Three Unequally Spaced Signals as Function of 1/K = δ/ρ
K=10, Maximum Limit
8
6
1.25
9
4
7
1.5
5
2
3
1
Single Pulse Maximum ρ/2π = 4×10^6
10^6
10^5
Measured ρ/2π = 4 Mc
Minimum Power
P/Pcw
24
22
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
δ/ρ = 1/K

Fig. 14
Single Pulse and its Equivalent Rectangular Pulse Used in the Determination of Single Pulse Breakdown

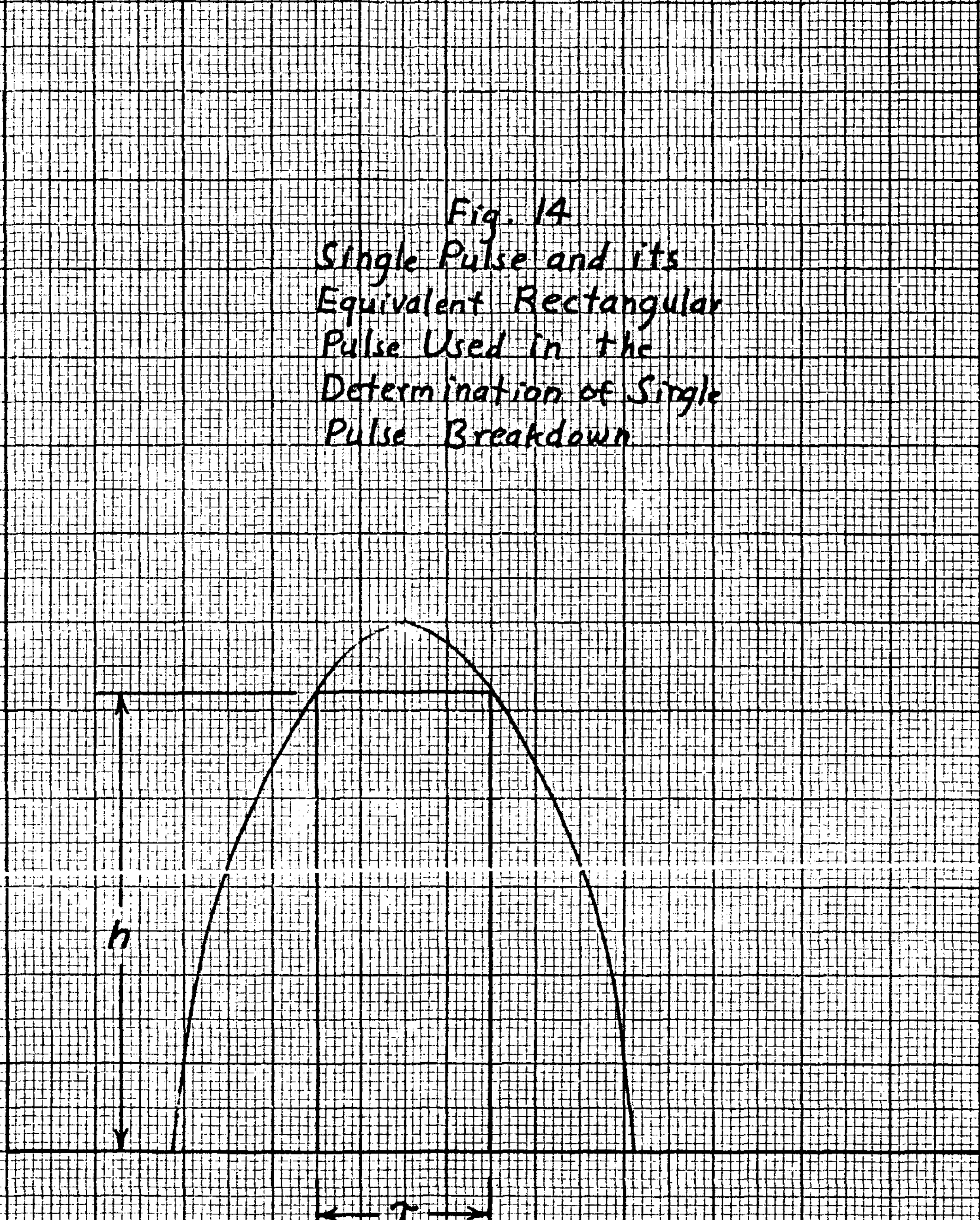


Fig. 15
Measuring System

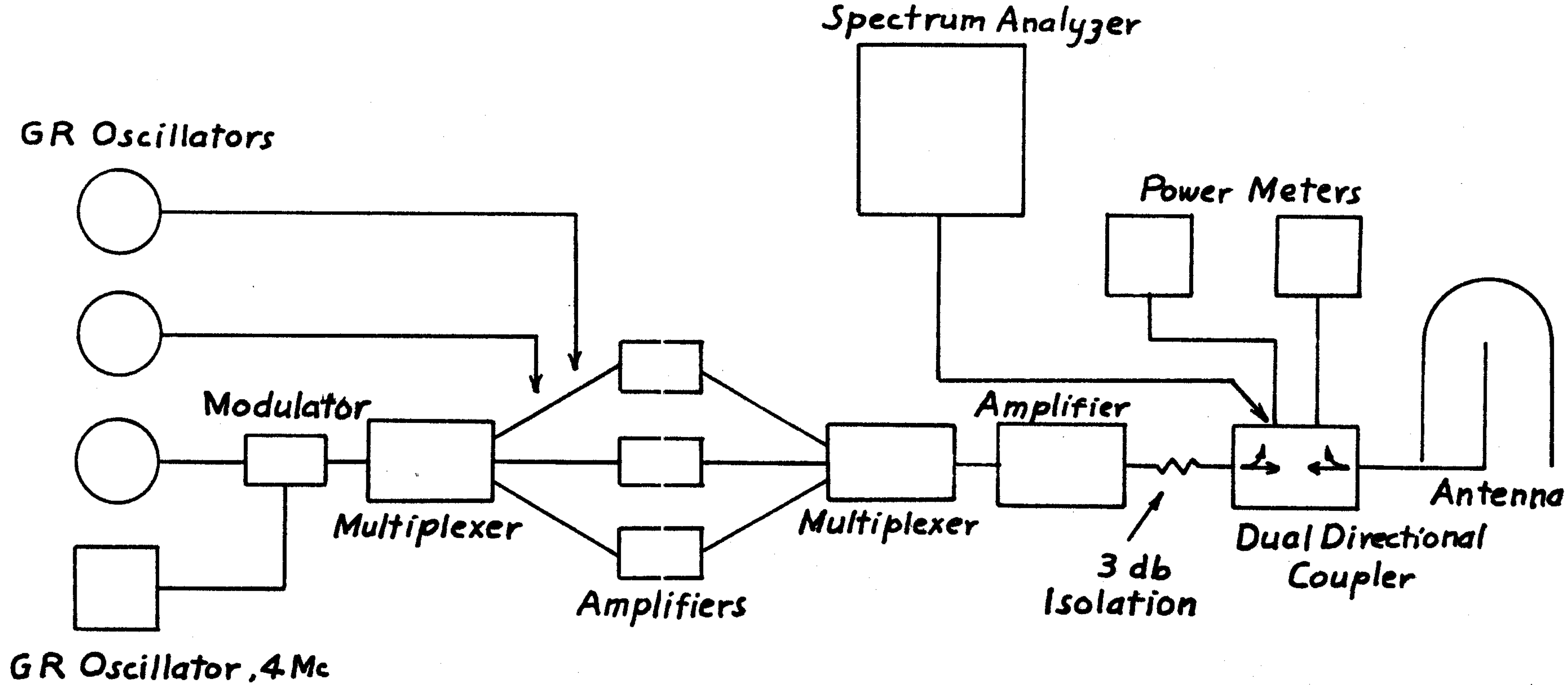


Fig. 16
Measured Antenna Power
for Breakdown as Function
of Frequency

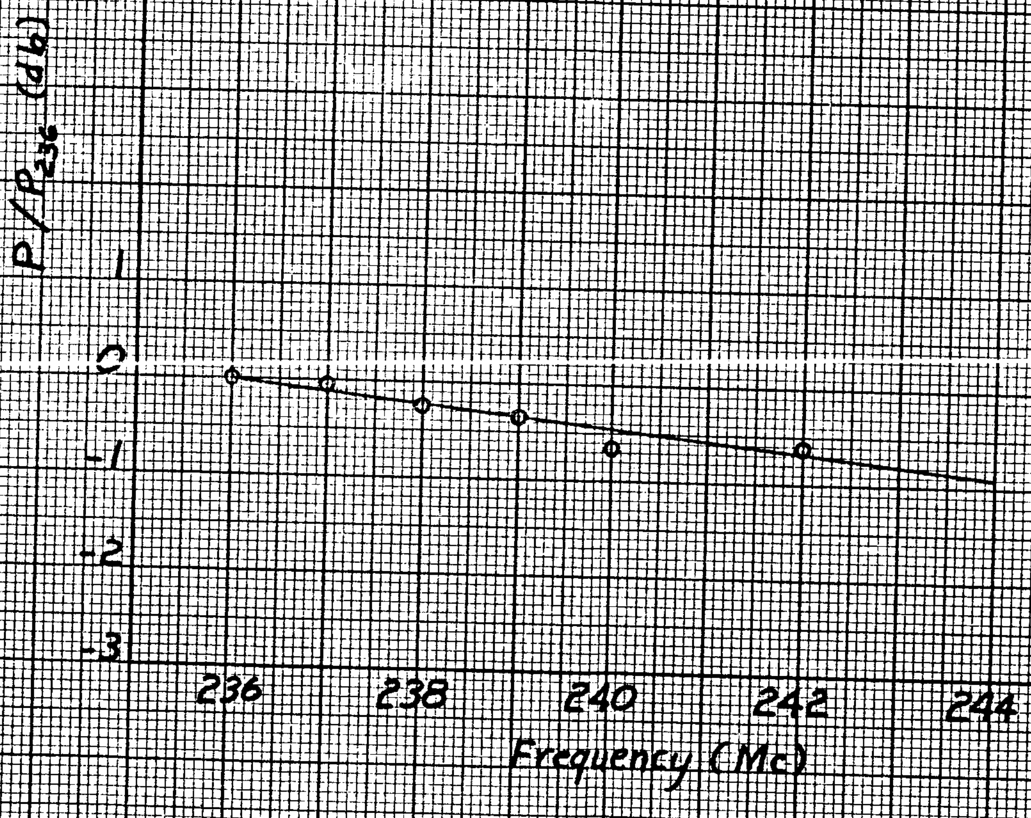
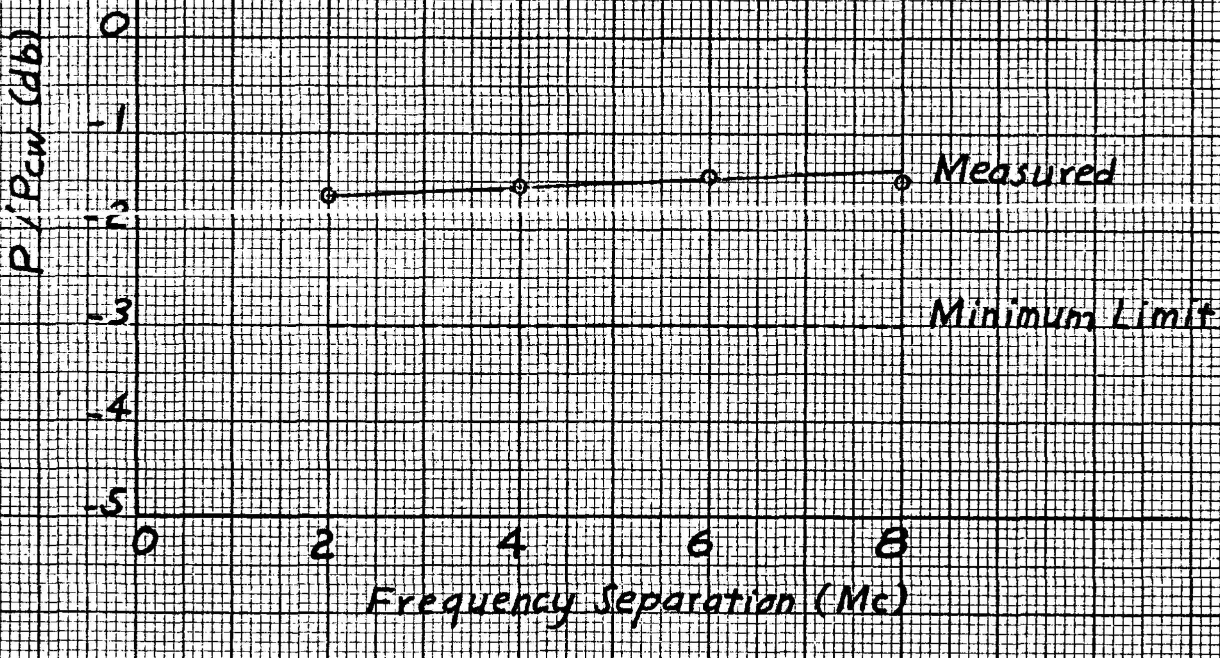


Fig. 17
Two-Signal Breakdown
as Function of
Frequency Separation
Lowest Frequency 236 Mc



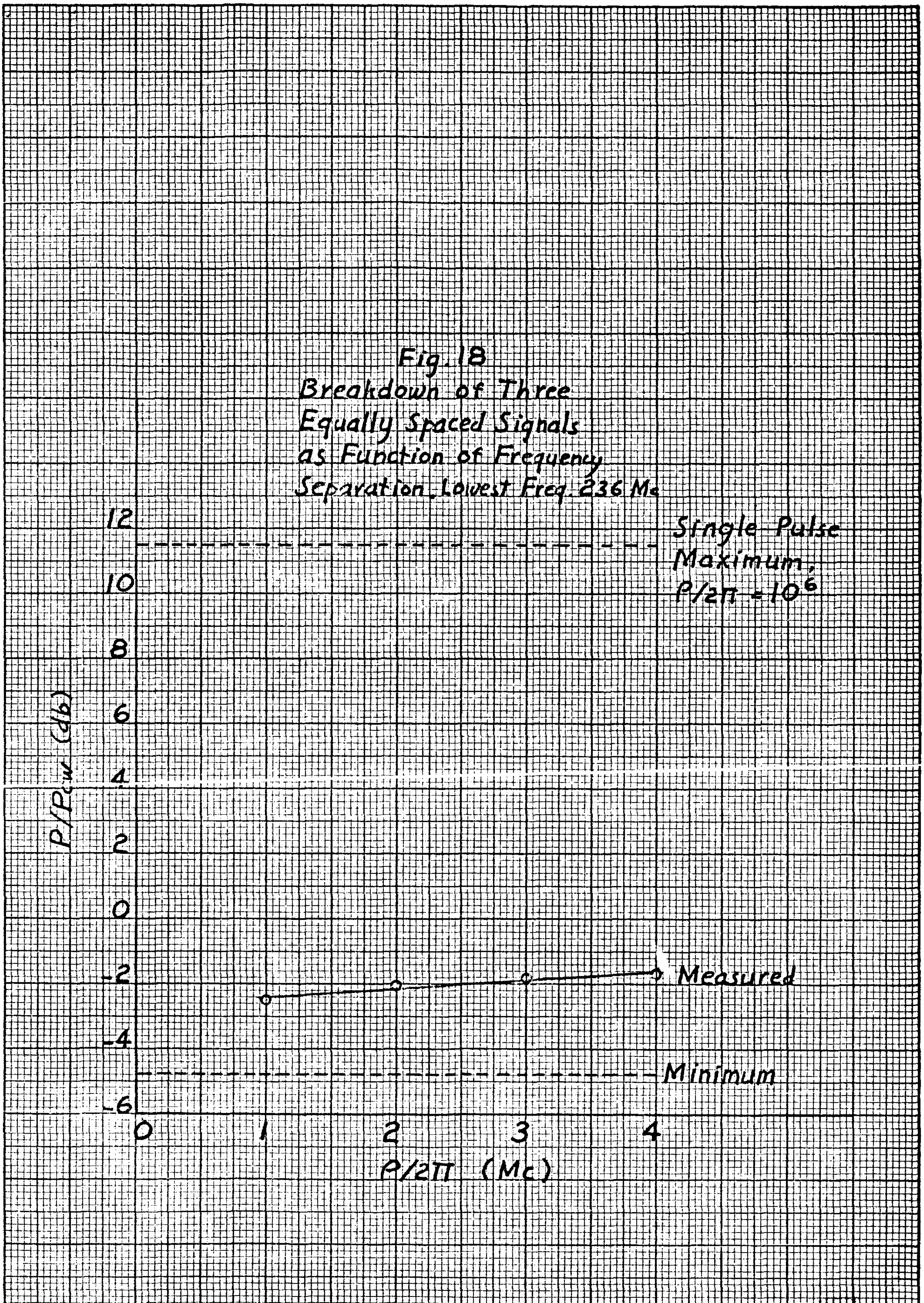


Fig. 18
Breakdown of Three
Equally Spaced Signals
as Function of Frequency
Separation. Lowest Freq. 236 Mc
Single Pulse
Maximum,
P/2π = 10^6
Measured
Minimum
P/Pcw (db)
12
10
8
6
4
2
0
-2
-4
-6
0
1
2
3
4
P/2π (Mc)

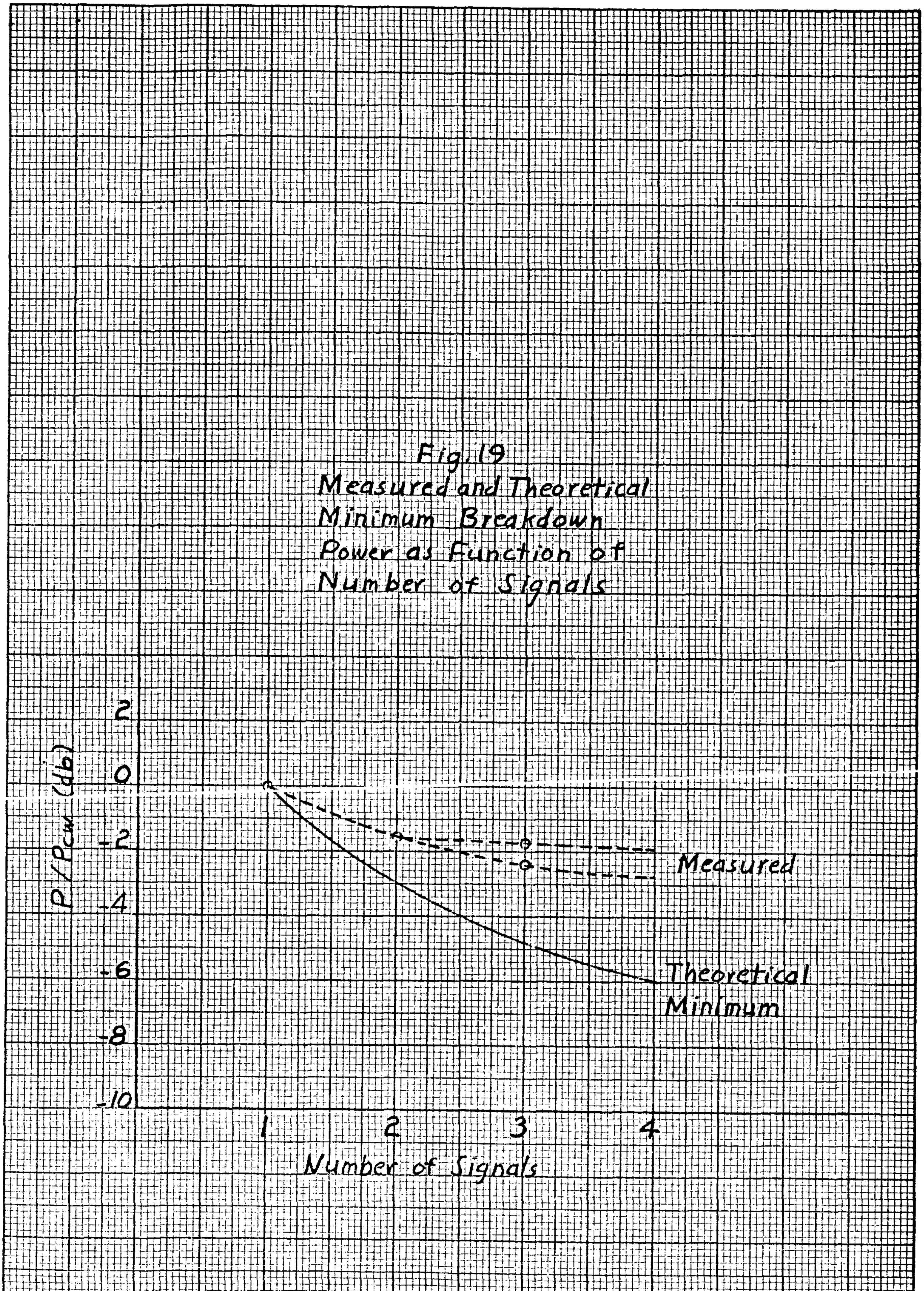


Fig. 19
Measured and Theoretical
Minimum Breakdown
Power as Function of
Number of Signals
P/Pcw (db)
2
0
-2
-4
-6
-8
-10
1
2
3
4
Number of Signals
Measured
Theoretical
Minimum